8.1.

The pyroelectric response in the framework of the problem can be found as

$$p = \frac{D_3}{\Delta T}$$

To find the relation between D_3 and ΔT, we will use the constitutive equations:

$$D_i = \varepsilon_0 K_{ij} E_j + d_{ij}\sigma_j + p_i \Delta T,$$

$$\varepsilon_i = d_{ji} E_j + s_{ij}\sigma_j + \alpha_i \Delta T.$$

In both cases, the electric field $E_3 = 0$ since the (001) electrodes are electrically connected. In order to simplify the equation for D_3, we use the $4mm$ symmetry restrictions for tensor K_{ij} ($K_{31} = K_{32} = 0$), thus $K_{31}E_1 = K_{32}E_2 = 0$. The equation for D_3 attains the following form:

$$D_3 = d_{3j}\sigma_j + p_3 \Delta T$$

In case **(a)**, the sample is mechanically free, implying all $\sigma_j = 0$. Then, $D_3 = p_3 \Delta T$, and

$$p_{(a)} = \frac{D_3}{\Delta T} = p_3.$$

In case **(b)**, the sample is kept mechanically free in x_1 and x_2 directions, implying $\sigma_1 = \sigma_2 = \sigma_4 = \sigma_5 = \sigma_6 = 0$, and $\sigma_3 \neq 0$. Then, equation for D_3 rewrites as

$$D_3 = d_{33}\sigma_3 + p_3 \Delta T$$

To find σ_3, we use the constitutive equation for $\varepsilon_3 = 0$, which must not change during the measurement (note that $E_3 = 0$):

$$\varepsilon_3 = d_{j3}E_j + s_{33}\sigma_3 + \alpha_3 \Delta T = d_{13}E_1 + d_{23}E_2 + s_{33}\sigma_3 + \alpha_3 \Delta T$$

Having applied the symmetry restrictions for $4mm$ point group on the piezoelectric tensor, which have the form (see Symmetry Tables)

$$d = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix},$$

$$\varepsilon_3 = s_{33}\sigma_3 + \alpha_3 \Delta T = 0 \quad \Rightarrow \quad \sigma_3 = -\frac{\alpha_3}{s_{33}}\Delta T,$$

$$D_3 = \left(p_3 - \frac{d_{33}\alpha_3}{s_{33}}\right)\Delta T,$$

$$p_{(b)} = \frac{D_3}{\Delta T} = p_3 - \frac{d_{33}\alpha_3}{s_{33}}.$$

Thus, in **(a)** and **(b)** the measured pyroelectric responses are different. Specifically,

$$p_{(a)} - p_{(b)} = \frac{d_{33}\alpha_3}{s_{33}}$$

8.2

The capacitance in the framework of the problem can be found as

$$C = \frac{\Delta Q}{\Delta V} = \frac{\Delta D_3 \cdot S}{\Delta E_3 \cdot L},$$

where ΔV is the change of applied voltage, ΔQ is the change of the charge on the electrodes.

To find the relation between D_3 and E_3, we will use the constitutive equations at constant temperature:

$$D_i = \varepsilon_0 K_{ij} E_j + d_{ij}\sigma_j,$$

$$\varepsilon_i = d_{ji} E_j + s_{ij}\sigma_j.$$

Applying the $4mm$ symmetry restrictions for tensor K_{ij} ($K_{31} = K_{32} = 0$), one can rewrite the equation for D_3 as follows:

$$D_3 = \varepsilon_0 K_{33} E_3 + d_{3j}\sigma_j.$$

In case **(a)**, the sample is mechanically free, implying all $\sigma_j = 0$. Then, $D_3 = \varepsilon_0 K_{33} E_3$, and

$$C_{(a)} = \frac{D_3 \cdot S}{E_3 \cdot L} = \varepsilon_0 K_{33} \frac{S}{L}.$$

In case **(b)**, the sample is kept mechanically free in x_1 and x_2 directions, implying $\sigma_1 = \sigma_2 = \sigma_4 = \sigma_5 = \sigma_6 = 0$, and $\sigma_3 \neq 0$. Then, equation for D_3 rewrites as

$$D_3 = \varepsilon_0 K_{33} E_3 + d_{33}\sigma_3.$$

To find σ_3, we use the constitutive equation for $\varepsilon_3 = 0$, which must not change during the measurement:

$$\varepsilon_3 = d_{j3} E_j + s_{33}\sigma_3 = d_{13}E_1 + d_{23}E_2 + d_{33}E_3 + s_{33}\sigma_3$$

Having applied the symmetry restrictions for $4mm$ point group on the piezoelectric tensor, which have the form (see Symmetry Tables)

$$d = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix},$$

$$\varepsilon_3 = d_{33}E_3 + s_{33}\sigma_3 = 0 \quad \Rightarrow \quad \sigma_3 = -\frac{d_{33}}{s_{33}} E_3,$$

$$D_3 = \left(\varepsilon_0 K_{33} - \frac{d_{33}^2}{s_{33}}\right) E_3,$$

$$C_{(b)} = \frac{D_3 \cdot S}{E_3 \cdot L} = \left(\varepsilon_0 K_{33} - \frac{d_{33}^2}{s_{33}}\right)\frac{S}{L}.$$

Thus, in **(a)** and **(b)** the measured capacitances are different. Specifically,

$$\frac{C_{(a)} - C_{(b)}}{C_{(a)}} = \frac{d_{33}^2/s_{33}}{\varepsilon_0 K_{33}} = 0.35$$